BIOT - SAVART'S LAW

"Fundamental law that gives the magnetic field produced by a steady or (constant) current."

For a current I flowing in/along a curve traced out by $d\vec{s}$ at a point in space defined by the position vector $\vec{r}$, the magnetic field is;

$$\boxed{\bar{B} = \int d\bar{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\bar{s} \times \bar{r}}{r^2}}$$

where

μ_0 = Permeability of free space, $\mu_0 = 4\pi \times 10^{-7}$ T-m/A.

The integral is a contour integral along the circuit in the direction of flow of current.

Important conclusions.

(1) $d\bar{B}$ is perpendicular to both $d\bar{s}$ and $\bar{r}$

(2) $|d\bar{B}|$ is inverse square law

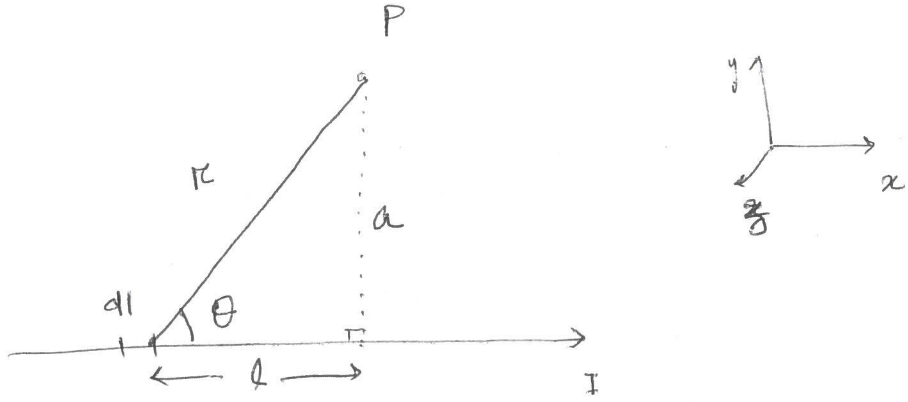
(3) $|dB|$ is proportional to current and to $|ds|$

(4) $|dB|$ is proportional to θ $(\sin\theta)$ which is the angle b/w $d\bar{s}$ and $\bar{r}$

Example - 1

Find the field due to a straight current carrying wire.

Soln



Direction of the field at P due to a current element

$\vec{dl} \quad = \quad \bar{dl} \times \bar{r}$

vector out of the page.

So;

$$\frac{\bar{dl} \times \bar{r}}{r^2} = \frac{|dl \sin\theta|}{r^2} \hat{k}$$

a: distance b/w the point P from the wire.

$$r = \frac{a}{\sin\theta}.$$

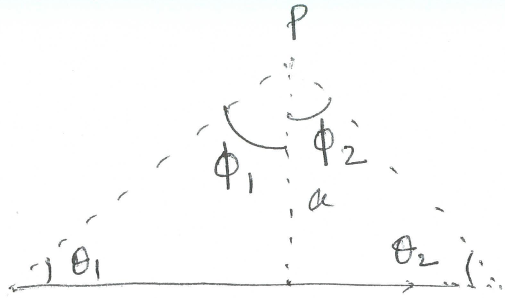
$$l = a\cot\theta$$

$$\Rightarrow dl = -\frac{a}{\sin^2\theta} d\theta$$

So,

$$\frac{\bar{dl} \times \bar{r}}{r^2} = \frac{\sin\theta}{a} d\theta \, \hat{k}$$

Since the magnetic fields due to all current elements at P are parallel to z-direction, the ends of the wire subtent angle ϕ_1 and ϕ_2.



So, $\overline{B} = \dfrac{\mu_0 I}{4\pi a} \displaystyle\int_{\theta_1}^{\theta_2} \sin\theta \, d\theta \, \hat{k}$

Notice θ to ϕ here.

$$= \frac{\mu_0 I}{4\pi a} \left[\sin\phi_1 + \sin\phi_2 \right] \hat{k}$$

For an infinite long wire, $\phi_1 = \phi_2 = \frac{\pi}{2}$.

So, $\boxed{\vec{B} = \dfrac{\mu_0 I}{2\pi a} \hat{k}}$