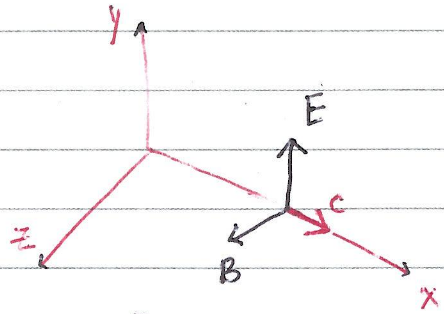
DERIVATION OF EM. WAVES FROM MAXWELL'S EQUATIONS

Assumption :

1) Electric field (E) is in y-direction.
2) Magnetic field (B) is in z-direction
3) Both are functions of x and t.

$$\bar{E}(x,t) = E(x,t)\,\hat{j}$$
$$\bar{B}(x,t) = B(x,t)\,\hat{k}$$



Free space.

Deriving Maxwell's equation in free space.

① Curl of $\bar{E}$

$$\bar{\nabla} \times \bar{E}(x,t) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E(x,t) & 0 \end{vmatrix} = \frac{\partial}{\partial x} E(x,t)\,\hat{k}$$

By Faraday's law :-

$$\boxed{\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}} \quad -(1)$$

⇒ When E field varies spatially, it manifests a time-varying B-field.

② Curl of $\bar{B}$

$$\bar{\nabla} \times \bar{B}(x,t) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & B(x,t) \end{vmatrix} = -\frac{\partial B}{\partial x}\,\hat{j}$$

By Ampere's correction to Maxwell's law:-

$$\boxed{\frac{\partial B}{\partial x} = -\mu_0 \epsilon_0 \frac{\partial E}{\partial t}} \quad -(2) \Rightarrow$$ Spatial variation of B manifests time variation of E field

Taking eq^n 1

$$\frac{\partial}{\partial x}\left(\frac{\partial E}{\partial x}\right) = -\frac{\partial}{\partial x}\left(\frac{\partial B}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\frac{\partial B}{\partial x}\right)$$

$$= -\frac{\partial}{\partial t}\left(-\mu_0 \epsilon_0 \frac{\partial E}{\partial t}\right)$$

$$= \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

So, $$\boxed{\frac{\partial^2 E}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}} \quad -(3)$$

Taking eq^n 2

$$\frac{\partial}{\partial x}\left(\frac{\partial B}{\partial x}\right) = \frac{\partial}{\partial x}\left(-\mu_0 \epsilon_0 \frac{\partial E}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\mu_0 \epsilon_0 \frac{\partial E}{\partial x}\right)$$

$$= \frac{\partial}{\partial t}\left(\mu_0 \epsilon_0 - \frac{\partial B}{\partial t}\right)$$

$$= \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2}$$

So, $$\boxed{\frac{\partial^2 B}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2}} \quad -(4)$$

Eq^n (3), (4) are general wave equations, travelling in x-direction

$$\frac{\partial^2 \varphi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \varphi}{\partial t^2}$$

Equating coefficients, we get c = speed of e.m. wave

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 2.99 \times 10^8 \text{ m/sec}.$$

The simplest solutions for (3), (4) are :-

$$\hat{E}(x,t) = E_0 \cos(kx - \omega t)$$

$$\vec{B}(x,t) = B_0 \cos(kx - \omega t)$$

$k = \frac{2\pi}{\lambda}$ ⇒ wave number

$\omega = 2\pi f$ ⇒ angular frequency.

Debi prasad Pattnaik.